INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - III, Statistics - IV, Final Examination, May 9, 2019
Time: $2\frac{1}{2}$ hours

1. Let $X_1, \ldots, X_n$ be a random sample from a continuous distribution with c.d.f. F and density f, both of which are completely unknown.
(a) Define the moving bin average estimate f_n of f.
(b) Show that $f_n(x) \longrightarrow f(x)$ in probability as $n \longrightarrow \infty$ at each x if the bin width in (a) is proportional to $n^{-1/3}$. [10]

2. Let $X_1, \ldots, X_n$ be a random sample from the Exponential distribution with mean $1/\lambda$, $\lambda > 0$. Under the prior, let λ have the Exponential distribution with mean 1.
(a) Find the HPD estimate of λ.
(b) Find the posterior mean of λ.
(c) Show how to construct 95% HPD credible interval for λ. [15]

3. Suppose $X_1, \ldots, X_k$ and $Y_1, \ldots, Y_l$ are independent random samples from $N(\theta, \sigma^2)$ and $N(\theta, 2\sigma^2)$, respectively, where $-\infty < \theta < \infty$ is unknown but $\sigma^2 > 0$ is known. Derive the minimax estimator of θ under the squared error loss. [12]

4. (a) Show that the risk set in an S-game is a closed convex set if the action space $\mathcal{A}$ is a finite set.
(b) Show that, in an S-game, any strategy δ^* for player II is an admissible strategy if its risk point is a lower boundary point of the risk set S.
(c) Can a minimax estimator be inadmissible in a statistical problem? Justify your answer. [13]